Modelling the real exchange rate between the U.S. and Germany

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Purchasing power parity (PPP) theory states that the exchange rate between two countries should exactly reflect the ability to buy goods in each country. Nonetheless, variations exist and Edward O'Brien sets out to model the real exchange rate and see whether PPP holds over longer time periods. Using co-integration and regression, he concludes that PPP fails to hold.

Introduction

The aim of this paper is to model the real exchange rate between the U.S. and Germany. Theory proposes that this real exchange rate should be equal to one. Therefore, the nominal exchange rate should be equal to the relative price level of the two countries. If this theory holds, the series being analysed should co-integrate, and have regression coefficients of plus and minus one.

I intend to investigate if this is in fact the case, by analysing the following series, over the period 1960 to 1998:

- The nominal exchange rate between the U.S. and Germany
- The price index for Germany
- The price index for the U.S.

The paper is organised as follows, with the necessary theory supplied as required:

- Theory of Exchange Rates and Purchasing Power Parity.
- Econometric Theory.
- Methodology and Discussion.
- Conclusion.

It should be noted that all data used was taken from *International Financial Statistics*, and analysed using *E-Views-3*.

Theory of Exchange Rates and Purchasing Power Parity

The theory of Purchasing Power Parity (PPP) states that the exchange rate between two countries' currencies equals the ratio of price levels in those countries. This theory of PPP is more formally known as *absolute* Purchasing Power Parity. The theory of PPP, be it absolute or relative, can be expressed mathematically. Let P_{US} be the Dollar price of a reference commodity basket sold in the U.S., and P_{DM} be the Deutschmark price of the same basket of goods in Germany. The theory of PPP predicts a Dollar/Deutschmark exchange rate of:

$$E_{t\ US/DM} = P_{t\ US} / P_{t\ DM} \quad (1)$$

In equation 1, E_t represents the nominal exchange rate, and P_t the respective price levels. The *nominal exchange rate* is defined as the exchange rate between one currency and another, in this case Dollars and Deutschmarks. The theory of PPP, is based on the Law of One Price, which assumes that the real exchange rate is equal to one. The Law of One Price states that in competitive markets, free of transportation costs and official barriers to trade, identical goods sold in different countries must sell for the same price, when their prices are expressed in terms of the same currency (Krugman & Obstfeld, 2000). The *real exchange rate* is defined as the exchange rate between two currencies, divided by the ratio of the price levels in the two countries. If we let R_t be the real exchange rate, equation 1 becomes:

$$E_{t\ US/DM} = R_t\,(P_{t\ US} / P_{t\ DM}) \quad (2)$$

Given the theory that $R_t = 1$, equation 1 and 2 are identical. However, the assumptions of the theory of PPP do not hold under empirical analysis. The reasons for this poor performance are obvious. The assumptions of the theories of PPP and the Law of One Price fail to hold in reality:

- Contrary to the assumptions of the Law of One Price, transportation costs and official barriers to trade do exist.
- Markets are not perfectly competitive.
- The commodity baskets measured in each country differ, and therefore, the price levels reported will not be comparable.

The *Big Mac* index is commonly used as a measure of PPP. This index shows that no two countries have a real exchange rate equal to one. In 1998, this index showed a price difference between Germany and the U.S. of $0.13 for a *Big Mac*.[1] Although this difference is small, it should be zero under the theory of PPP. So in general, one would not expect R_t, the real exchange rate, to equal 1. In fact, it should be equal to some constant value. Rearranging equation (2), the real exchange rate, R_t, can be expressed as:

[1] "The Hamburger Standard." *The Economist*. (April 11, 1998) p. 58.

$$R_t = E_{t\ US/DM} (P_{t\ DM} / P_{t\ US}) \qquad (3)$$

This paper aims to investigate these theories. Does the theory of PPP hold, or shall the empirical evidence be borne out?

Econometric Theory

Spurious regression can be a particular problem with time series data. This is where the regression results initially look favourable, but upon further investigation, become increasingly suspect. However, if two time series are co-integrated, regression results may not be spurious. To quote Granger, "A test for co-integration can be thought of as a pre-test to avoid 'spurious regression' situations"(Granger, 1986:226). A stochastic process is said to be *stationary* if its mean and variance are constant over time and the value of covariance between two time periods depends only on the distance or lag between the two time periods and not on the actual time at which the covariance is computed (Gujarati, 1995). If a time series is not stationary as defined above, it is a *non-stationary* time series.

Given two or more series, that are non-stationary or random walk stochastic processes, the linear combination of these variables might be stationary. Although the series may be trending upwards in a stochastic fashion, they seem to be trending together. More formally, if the regression residual of two non-stationary series is found to be stationary, then those series are said to be co-integrated.

These theories can be applied directly to the problem of real exchange rates. Firstly, I am interested in the changes over time in exchange rates and price levels. Therefore I have taken the natural logarithm of equation (3). I have also included coefficients.

By taking the natural logarithm, and rearranging, this equation becomes:

$$\text{Ln } E_{t\ US/DM} = \beta + \beta_1 \text{Ln } P_{t\ US} - \beta_2 \text{Ln } P_{t\ DM} \qquad (4)$$

Note: Ln $R_t = \beta$, a constant, since R_t is itself a constant.

However, this equation is a statement of theory. For regression, we must include an error term also. This gives us:

$$\text{Ln } E_{t\ US/DM} = \beta + \beta_1 \text{Ln } P_{t\ US} - \beta_2 \text{Ln } P_{t\ DM} + U_t \qquad (5)$$

Given this information, this investigation must focus on two aspects.

1. If the series Ln $E_{t\ US/DM}$, Ln $P_{t\ US}$, and Ln $P_{t\ DM}$, are non-stationary, or I(1), and we find that U_t is stationary, or I(0), then we can say the series are co-integrated. If this is the case, any regression we carry out may not be 'spurious'.

By rearranging equation (5), U_t is:

$$U_t = \text{Ln } E_{t\ US/DM} - \beta - \beta_1 \text{ Ln } P_{t\ US} + \beta_2 \text{ Ln } P_{t\ DM} \qquad (6)$$

If U_t is stationary, it is said to be mean reverting. That is, it is not trending upwards or downwards over time.

2. In regressing the model given by equation (5), theory suggests that the coefficients will have an absolute value of 1. If this is so, equations (2), (3), and (4) will be valid, and theory of PPP will be correct. However, if these coefficients are significantly different from 1, it will suggest that this theory is invalid.

Therefore, I must test for co-integration, and then consider the regression results.

Methodology and Discussion.

The three series being considered can be found in the appendix A1 of this paper. The first operation was to take the natural logarithm of each observation of each of the three series. I then proceeded as follows:

Testing for Stationarity.

Stationarity can be tested by two methods. The first is a subjective visual inspection of the time series graph and its correlogram. The second is the unit root test. While I have included the time series graphs below, it is not easy to judge *by eye* if series are stationary or not.

Figure 1: A plot of the natural log of the nominal exchange rate between German Marks and US Dollars, over time

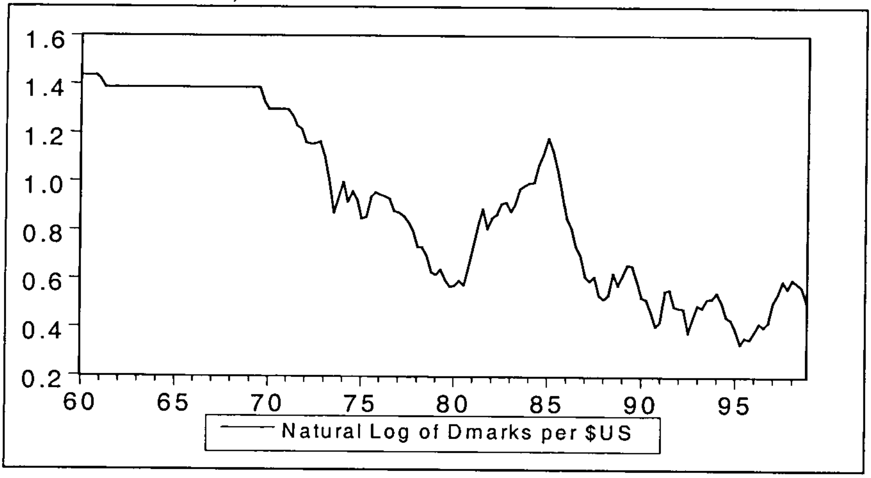


Figure 2: A plot of the natural log of the Consumer Price Index for Germany over time

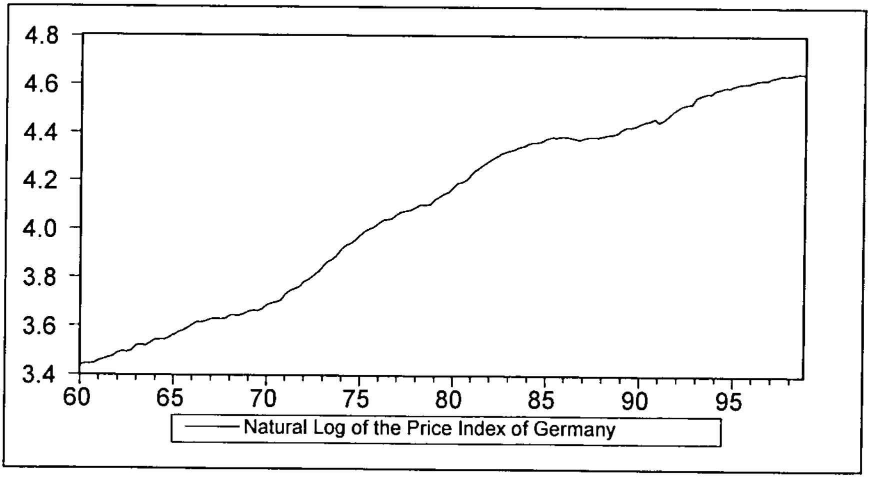
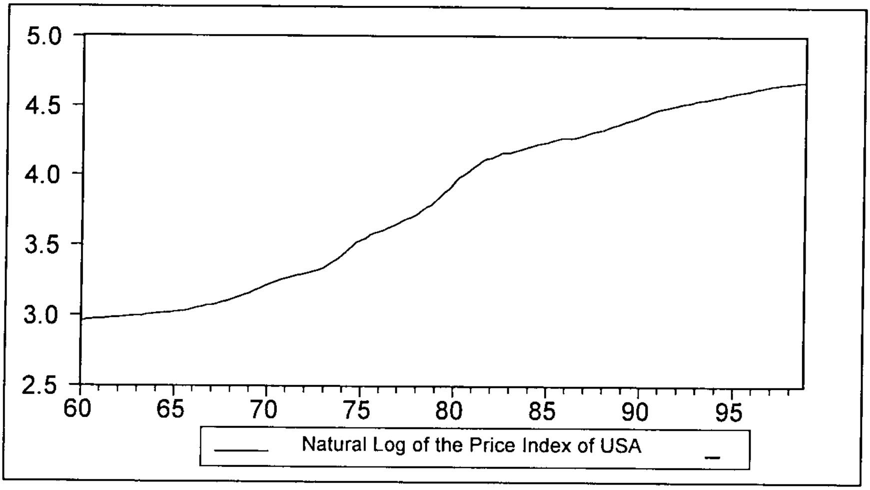


Figure 3: A plot of the natural log of the Consumer Price Index for the USA over time



Therefore, in testing for stationarity, I rely solely upon the unit root test. The unit root test is formulated as follows for the Augmented Dickey-Fuller (ADF) test:

$$\beta \text{ Ln } E_{t\ US/DM} = \beta \text{ Ln } E_{t-1\ US/DM} + \theta^{k}_{i=1} \beta_i \beta \text{ Ln } E_{t-1\ US/DM} + U_t \quad (7)$$

The hypothesis being tested is:

$$H_0: \beta = 1$$
$$H_1: \beta < 1$$

and the test statistic is:

$$t = \frac{\beta' - 0}{SE(\beta')}$$

Where β' is the estimate of β.

Using the ADF test, I determined the number of unit roots in the series. By testing first the *levels*, and then the *first differences* and so on, one can determine the number of unit roots. If the test fails to reject the test in levels, but rejects the test in first differences, then the series contains one unit root, and is integrated of order one, i.e. is I(1). In completing the ADF test, I included a constant term, but no linear trend, and allowed the number of lagged first difference terms, k, to be initially 6.

For the first series, Ln $E_{t\ US/DM}$, the ADF table for both level and first differences are shown below (Tables 1 & 2). As can be seen in the level, the null hypothesis of a unit root fails to reject against the one-sided alternative, since the t-statistic is not less than (does not lie to the left of) the critical value. However, for the first differences, the null hypothesis is rejected, even at the 1% critical value. Therefore, we can infer that Ln $E_{t\ US/DM}$ is integrated of order one, i.e. I(1). Ln $E_{t\ US/DM}$ is a non-stationary series.

Table 1: Unit Root Test of Ln $E_{t\ US/DM}$: Level

ADF Test Statistic	-1.134934	1% Critical Value*	-3.4752
		5% Critical Value	-2.8809
		10% Critical Value	-2.5770

Table 2: Unit Root Test of Y: First Differences

ADF Test Statistic	-3.994102	1% Critical Value*	-3.4755
		5% Critical Value	-2.8810
		10% Critical Value	-2.5770

Applying the same test procedure to the Ln $P_{t\ US}$, and Ln $P_{t\ DM}$ series yields similar results. Ln $P_{t\ US}$ is rejected at the 10% critical value, and Ln $P_{t\ DM}$ is rejected at the 1% level. They are both found to be non-stationary series. The ADF tables for both these series are included below (Tables 3, 4, 5 & 6).

Table 3: Unit Root Test of Ln $P_{t\ US}$: Level

ADF Test Statistic	-0.936737	1% Critical Value*	-3.4752
		5% Critical Value	-2.8809
		10% Critical Value	-2.5770

Table 4: Unit Root Test of Ln $P_{t\ US}$: First Differences

ADF Test Statistic	-2.651080	1% Critical Value*	-3.4755
		5% Critical Value	-2.8810
		10% Critical Value	-2.5770

Table 5: Unit Root Test of Ln $P_{t\ DM}$: Levels

ADF Test Statistic	-1.416407	1% Critical Value*	-3.4752
		5% Critical Value	-2.8809
		10% Critical Value	-2.5770

Table 6: Unit Root Test of Ln $P_{t\ DM}$: First Differences

ADF Test Statistic	-4.195920	1% Critical Value*	-3.4743
		5% Critical Value	-2.8805
		10% Critical Value	-2.5768

Note: The full ADF reports are available in appendix A2.

Co-integration and Regression

Now that we know the three series are all I(1), we must estimate the regression equation, and then test the residuals for stationarity. This will indicate whether the series co-integrate or not.

The regression equation is:

$$\text{Ln } E_{t\ US/DM} = \beta + \beta_1 \text{ Ln } P_{t\ US} - \beta_2 \text{ Ln } P_{t\ DM} + U_t \qquad (5)$$

The estimation output table (Table 7), is shown below, as is its' graph (Figure 4). It should be noted that the regression equation used in E-Views was:

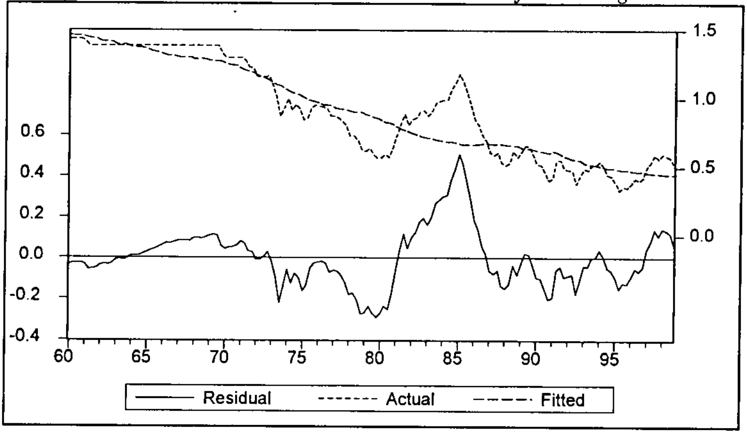
$$Y = C_1 + C_2X_{1t} + C_3X_{2t}.$$

This was done simply for convenience.

Table 7: The Estimated Regression Output

Dependent Variable: Y				
Method: Least Squares				
Date: 02/03/01 Time: 13:40				
Sample: 1960:1 1998:4				
Included observations: 156				
Y=C(1)+C(2)*X1+C(3)*X2				
	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	4.410775	0.463789	9.510300	0.0000
C(2)	0.028638	0.183868	0.155752	0.8764
C(3)	-0.881230	0.282791	-3.116183	0.0022
R-squared	0.844194	Mean dependent var		0.915815
Adjusted R-squared	0.842157	S.D. dependent var		0.360553
S.E. of regression	0.143246	Akaike info criterion		-1.029466
Sum squared resid	3.139461	Schwarz criterion		-0.970815
Log likelihood	83.29836	Durbin-Watson stat		0.097179

Figure 4: The Actual, Fitted, and Residual Plot for the Regression



For the co-integration regression, the value of R^2 is 0.844194. This is high and suggests a strong relationship between the variables.

The coefficient C (1) is simply the constant term. Coefficient C (2) is in fact β_1. Theory predicted its' value should be +1. As can be seen from above, its' actual value is 0.028638. The t-statistic value of 0.155752 fails to rejects the possibility that $\beta_1 = 0$. So one must assume a very real possibility that $\beta_1 = 0$. In fact one can be 87% sure that $\beta_1 = 0$.

The coefficient C (3) is actually β_2. Theory predicted its' value should be -1. As can be seen from above, its' actual value is -0.881230. The t-statistic value of -3.116183 rejects the possibility that $\beta_2 = 0$, even at the 1% level.

The Durbin-Watson test statistic reported is d = 0.097179. For 150 observations, with 2 explanatory variables, d_L = 1.598, and d_U = 1.651, at the 1% level of significance. Since d is less than d_L, we cannot reject positive serial correlation.

In order to establish the credibility of the multiple regression, it is worth carrying out single regressions, with the dependent variable regressed on each independent variable. The individual regressions tell a similar story, to that of the multiple regression. A regression of Ln $E_{t\ US/DM} = \beta + \beta_1$ Ln $P_{t\ US} + U_t$ gives the below results (Table 9).

Again, the coefficient, expected to equal +1, equals only 0.22. The value of R^2 is high, at -0.859970. These statistics do not mirror those of the multiple regression. However, the D-W statistic is such that positive serial correlation cannot be ruled out, which confirms the results of the multiple regression.

Table 9: Single Regression Output, Ln $E_{t\ US/DM} = \beta + \beta_1$ Ln $P_{t\ US} + U_t$.

Y=C(1)*X1				
	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.220533	0.010178	21.66694	0.0000

R^2 = -0.859970

The results from regressing Ln $E_{t\ US/DM} = \beta + \beta_2$ Ln $P_{t\ DM} + U_t$ are very similar. Here, the coefficient is just 0.22 also, with an R^2 of -0.499290. These results, quite different from those of the multiple regression, indicate a poor relationship between Ln $E_{t\ US/DM}$ and Ln $P_{t\ DM}$.

Table 10: Single Regression Output, Ln $E_{t\ US/DM} = \beta + \beta_2$ Ln $P_{t\ DM} + U_t$.

Y=C(1)*X2				
	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.214134	0.008602	24.89337	0.0000

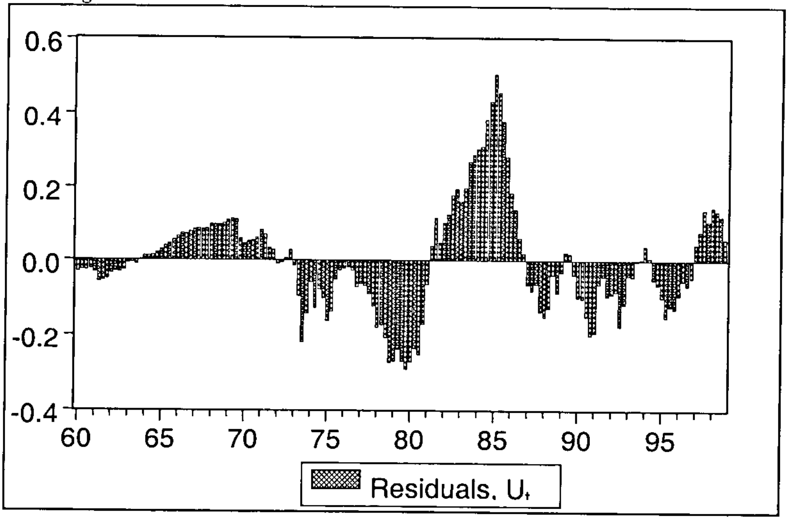
R^2 = -0.499290

Finally, multicollinearity was tested for by regressing, Ln $P_{t\ US}$ against Ln $P_{t\ DM}$. The results here are worrying. R^2 is a vey high 0.840701. This indicates a strong relationship between Ln $P_{t\ US}$ and Ln $P_{t\ DM}$, which will affect the multiple regression outcome. Having completed the co-integrated regression analysis, the next step is to evaluate the nature of the residuals.

The Residuals

Below is a graph of the regression residuals (Figure 5).

Figure 5: Residual Plot



As previously stated, if the series Ln $E_{t\ US/DM}$, Ln $P_{t\ US}$, and Ln $P_{t\ DM}$, are non-stationary, or I(1), and we find that U_t is stationary, or I(0), then we can say the series are co-integrated. Since it is now known that the three series are co-integrated, what remains is to test the residuals.

However, the ADF test as previously used, is only valid for unit root tests of a data series, and will be invalid if the series are based upon regression estimations. However, Engle and Granger have established a test for co-integration. It simply involves testing the stationarity of the regression residuals, but using a different set of critical values. In this context, the ADF test is known as the Augmented Engle-Granger (AEG) test.

The unit root test for regression residuals, or the co-integration test, is formulated as follows:

$$\beta U_t = (\beta - 1) U_{t-1} + \beta_t U_t = (\beta - 1) U_{t-1} + \theta_{j=1,2,\ldots,J} \beta_{t-j} \beta U_{t-j} + \beta_t \quad (8)$$

The hypothesis to be tested is:

$$H_0 : \beta = 1$$
$$H_1 : \beta < 1$$

And the test statistic is:

$$t\alpha = \frac{(p - 1)}{SE(p)}$$

Where p is the estimate of β.

The ADF, or AEG test statistic for the unit root of residuals is -3.049928. However, the critical value at the 1% level is approximately -5.0. This value is extrapolated from the values of appendix A4, since in this case, N=150, K=6. Note that N is the number of observations, and K is the number of lagged first difference terms. Although this value of -5.0 is approximate, it is so much larger than -3.049928, that the null hypothesis can be safely rejected. This implies the residuals are indeed stationary, or I(0), and that the series Ln $E_{t\ US/DM}$, Ln $P_{t\ US}$, and Ln $P_{t\ DM}$ are in fact co-integrated.

Conclusion

The focus of attention for the preceding section was twofold, concerning co-integration and regression. As introduced in early sections, the theory of PPP required that the three series concerned be co-integrated. I have shown that in fact these series are indeed co-integrated. Therefore, the regression results obtained for these time series should not be spurious.

Having established these facts, I then turned my attention to the regression. Initial results were mixed. While a high R^2 value was found, the regression coefficients were not +1 and -1, as theory would suggest. Upon further investigation, the regression results became doubtful. By regressing the dependent variable on the independent variables separately, the relationship seemed to fail. In the multiple regression a strong relationship was found to exist between Ln $E_{t\ US/DM}$ and Ln $P_{t\ DM}$. However, in the single regression, this relationship fails. The opposite is true of Ln$E_{t\ US/DM}$ and Ln$P_{t\ US}$.

The issues of multicollinearity and serial correlation were also considered. Results here indicate that multicollinearity is definitely a problem. The Durbin-Watson statistics for the regressions are such that I cannot reject the possibility of positive serial correlation in the residuals. Together, these results indicate that the multiple regression model is not a particularly good fit!

Given this information, what can I say about the initial model? Equation (4) is a statement of the theory of PPP. Since we now know that β_1 and β_2 do not equal unity in absolute value, we can say that the theory of PPP fails in this case. If $R_t = 1$, $\beta_1 = 1$, and $\beta_2 = -1$, then equation (4), below would simply revert back to equation (3), proving the theory of PPP.

$$Ln\ E_{t\ US/DM} = \beta + \beta_1 Ln\ P_{t\ US} - \beta_2 Ln\ P_{t\ DM} \qquad (4)$$

Finally, this paper began with an attempt to discover if R_t was in fact equal to one. In other words, is the theory of PPP valid? Having discussed the empirical evidence, one did not expect this theory to hold, but nevertheless, the analysis was undertaken. The final result was to show that for the case of the U.S. and Germany, between 1960 and 1998, the real exchange rate was not in fact one. The theory of PPP fails.

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